SIGGRAPH ASIA 2020 VIRTUAL
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MapTree: Recovering Multiple Solutions in the Space of Maps

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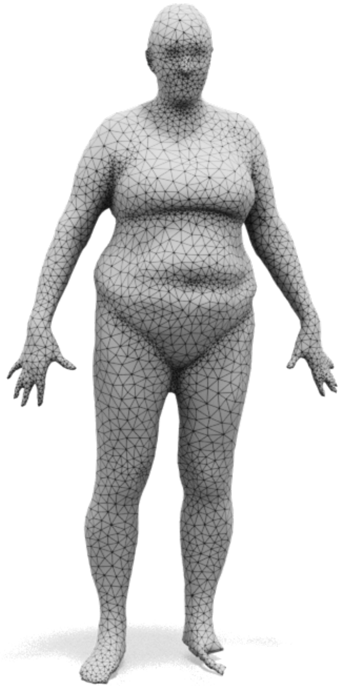
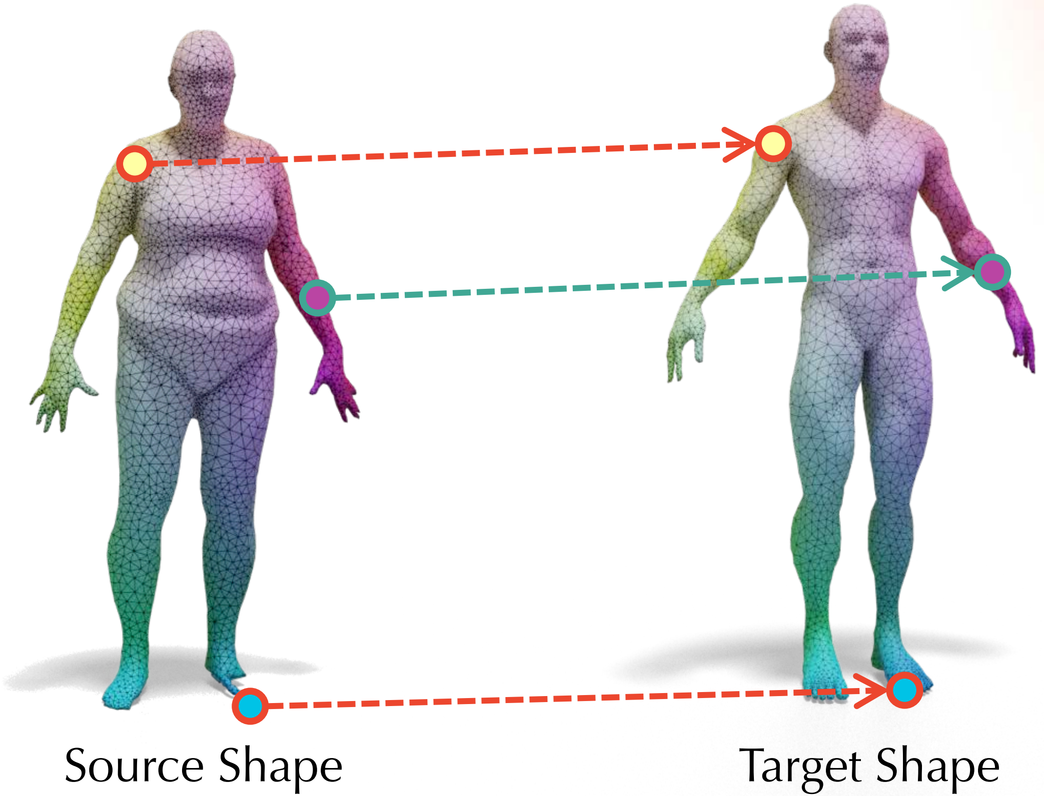
Map & Correspondences



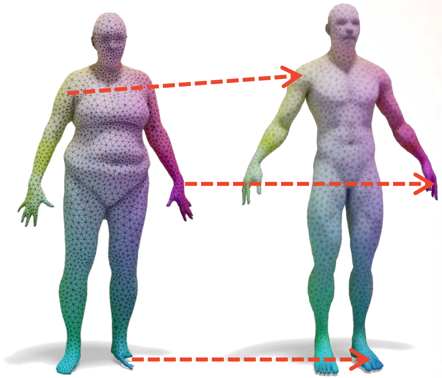
Triangle meshes

Map & Correspondences



Related work

- Point-based methods
  - [Bronstein et al. 2006],
  - [Huang et al. 2008]…
- Parametrization-based methods
  - [Lipman and Funkhouser 2009]
  - [Aigerman et al. 2017]…
- Optimal transport
  - [Solomon et al. 2016]
  - [Mandad et al. 2017]…
- Functional maps
  - [Ovsjanikov et al. 2012]
  - [Ezuz and Ben-Chen 2017]…

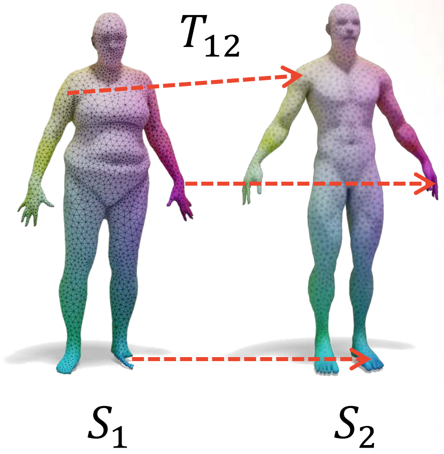


How to compute a map?

Problem Formulation

$$\min_{T_{12}} E(T_{12})$$

- Objectives $E(\cdot)$
  - geodesic distortion
  - Dirichlet energy
  - …
- Constraints
  - bijective map
  - …

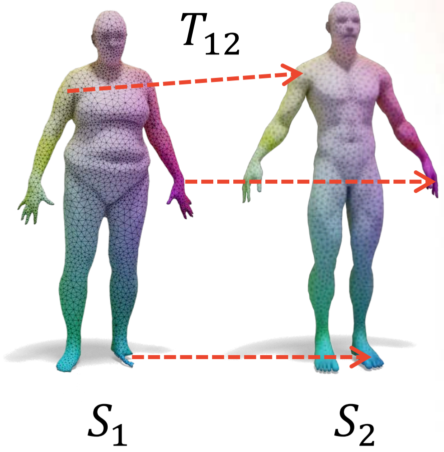


Complicated map space

Geodesic distortion

$$E(T_{12}) = \sum_{(v_i, v_j)} \left\| D_1(v_i, v_j) - D_2\left(T_{12}(v_i), T_{12}(v_j)\right) \right\|$$

- $D_k(v_i, v_j)$ stores the geodesic distance between the two vertices v_i and v_j on shape S_k



Complicated map space

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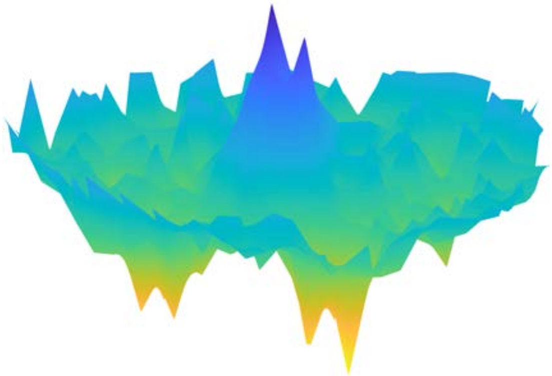
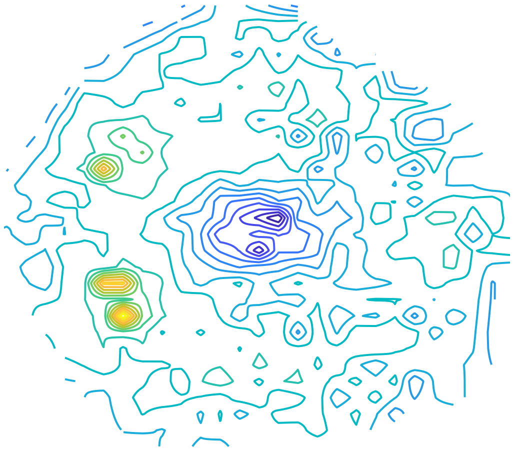


landscape



contour

Map space

- Discrete
- Not differentiable w.r.t. T_{12}
- Complicated $\mathcal{O}(n^n)$
- Multipe local-minima
- ……

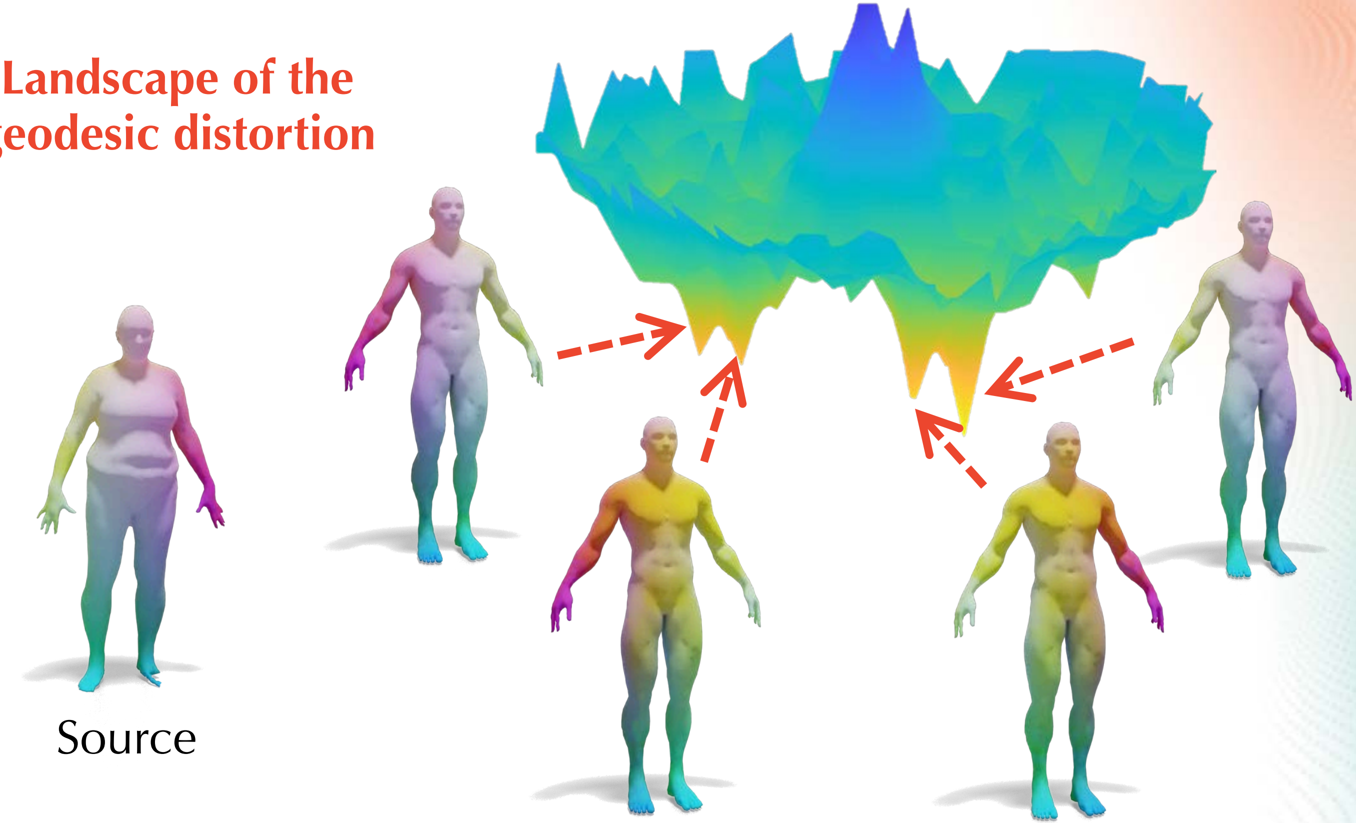
Avoid undesired local minima?

Existing solutions

- [Sung and Kim 2013] “Finding the M-best consistent correspondences between 3D symmetric objects”
    - For each vertex, find multiple correspondences candidates to resolve global symmetry ambiguity
- [Sahillioğlu and Yemez 2013] ″Coarse-to-fine Isometric Shape Correspondence by tracking symmetric flips″
    - Avoid symmetry flip during map computation

Find all meaningful maps!

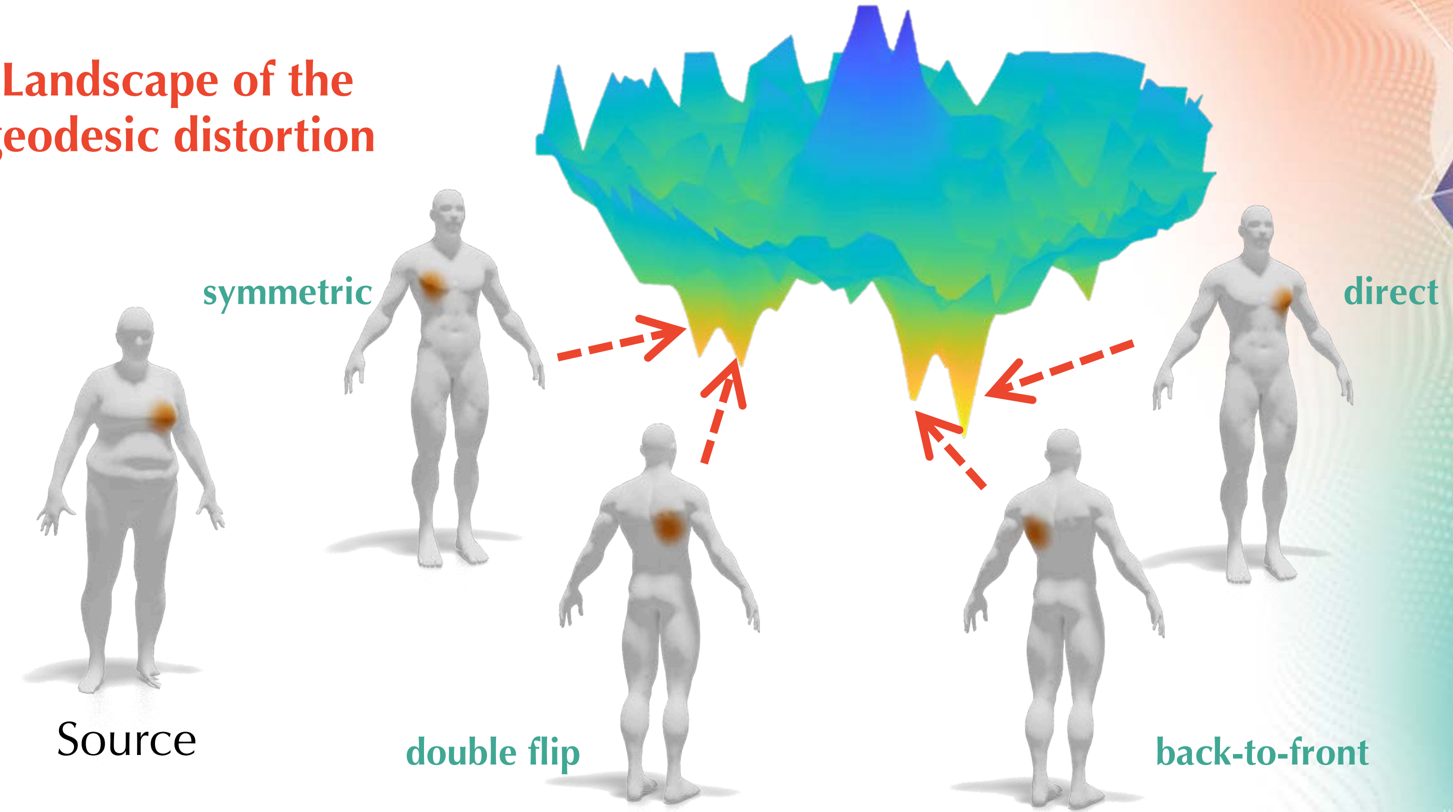
Landscape of the geodesic distortion



Source

Find all meaningful maps!

Landscape of the geodesic distortion



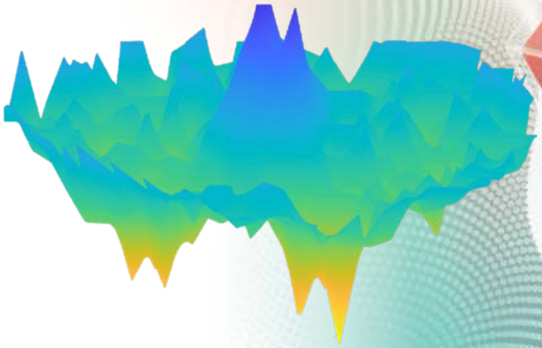
Find all meaningful maps!

Observations

- Intrinsic objectives: not discriminative w.r.t. self-symmetries
- Multiple ground-truth can exist!

Inspirations

- ~~Avoid symmetry flip during computation~~
- Find all meaningful maps and select later!
- Use Functional Map representation



Laplace-Beltrami Operator

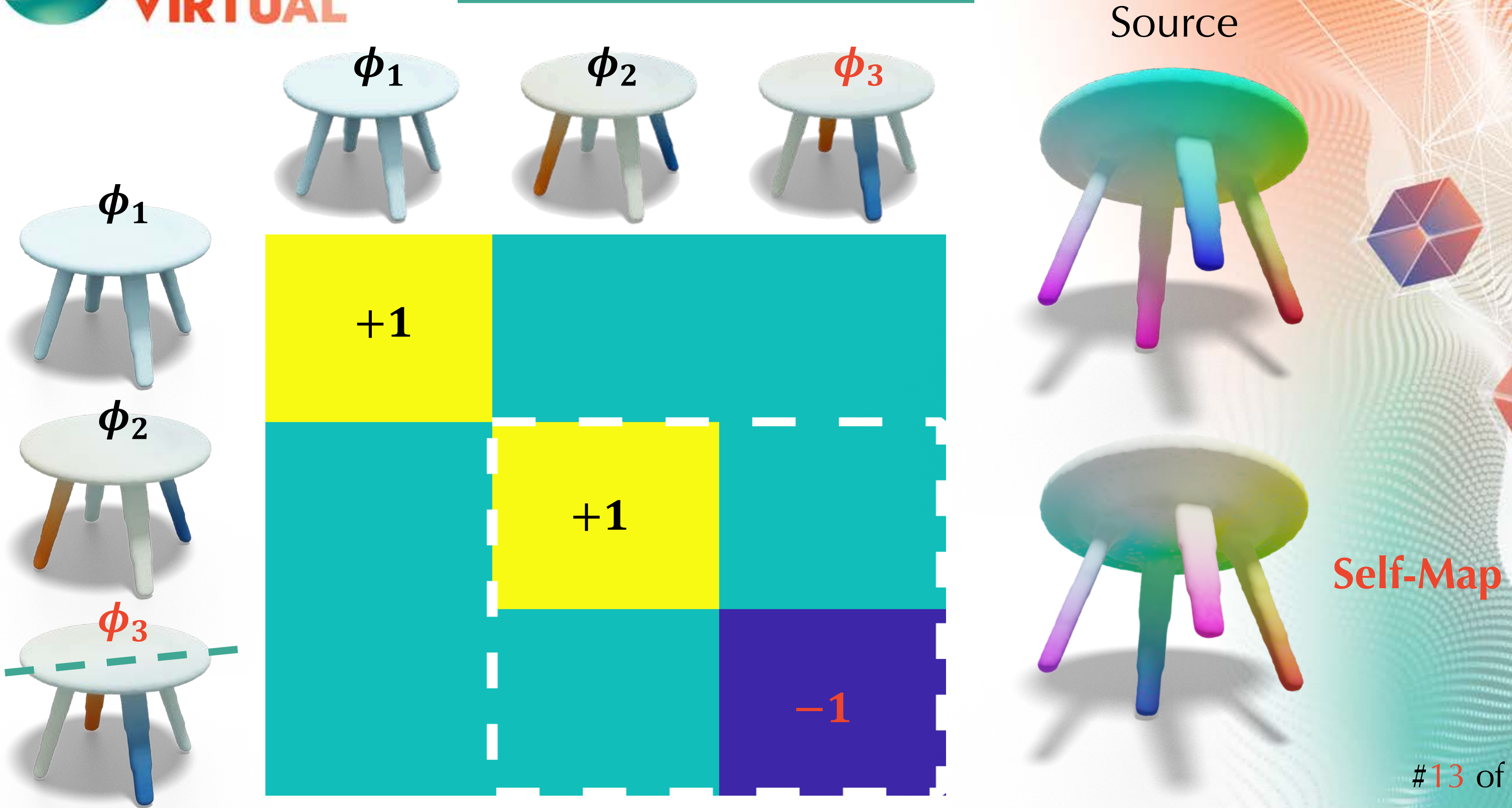
Helmholtz Equation

$$\Delta_S f = \lambda f$$

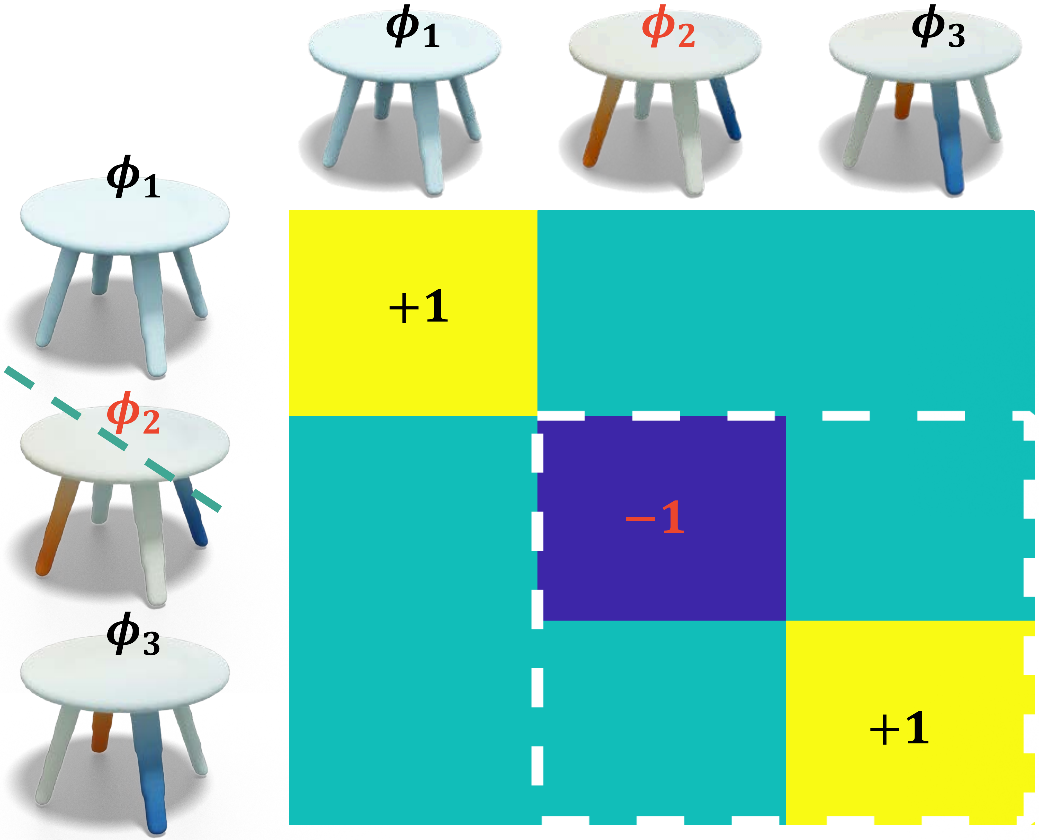
Shape S | ϕ_1 | ϕ_2 | ϕ_3 | ϕ_4

$0 = \lambda_1 \leq$ | $\lambda_2 \leq$ | $\lambda_3 \leq$ | $\lambda_4 \leq \cdots$

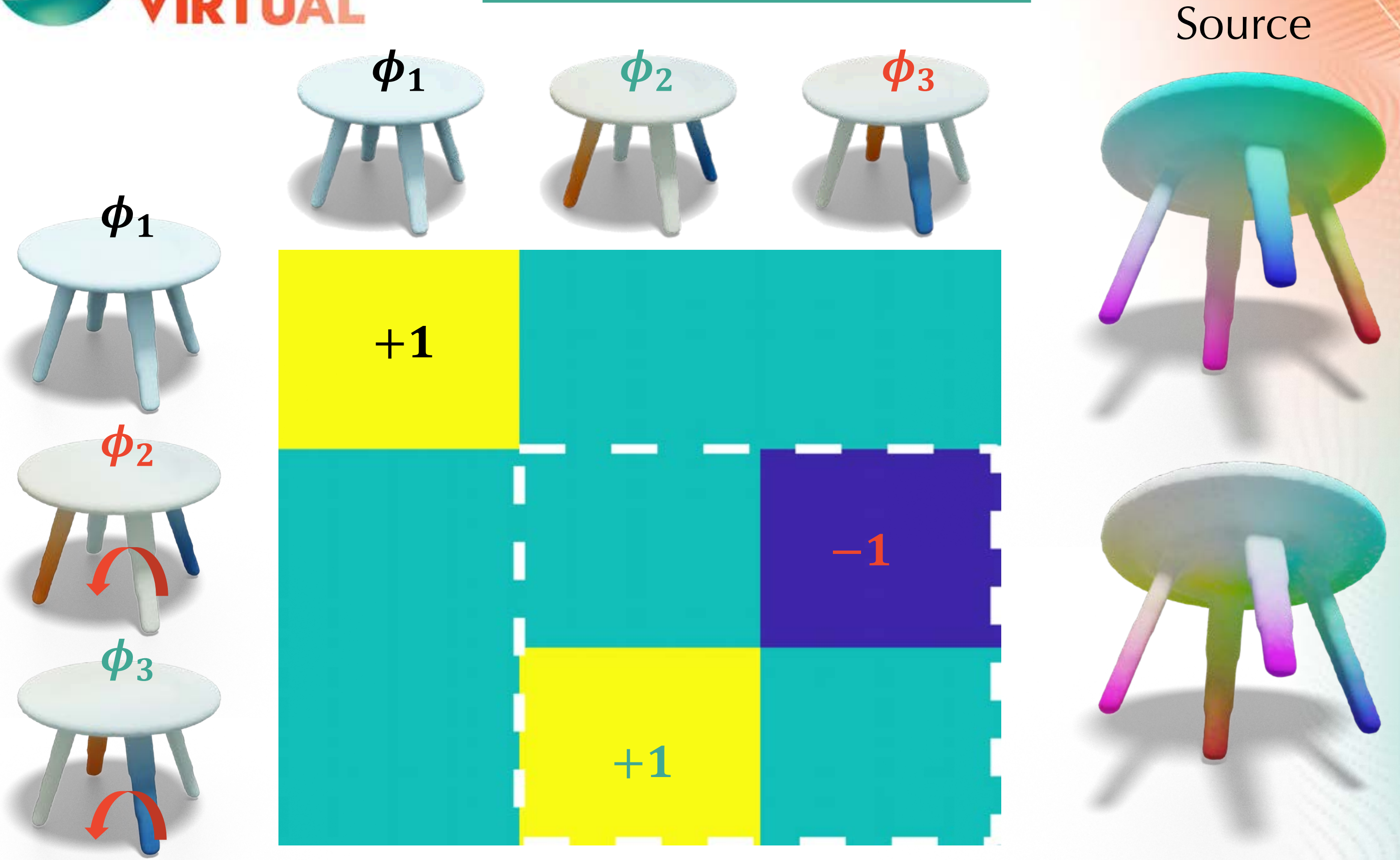
Functional map



Functional map



Functional map

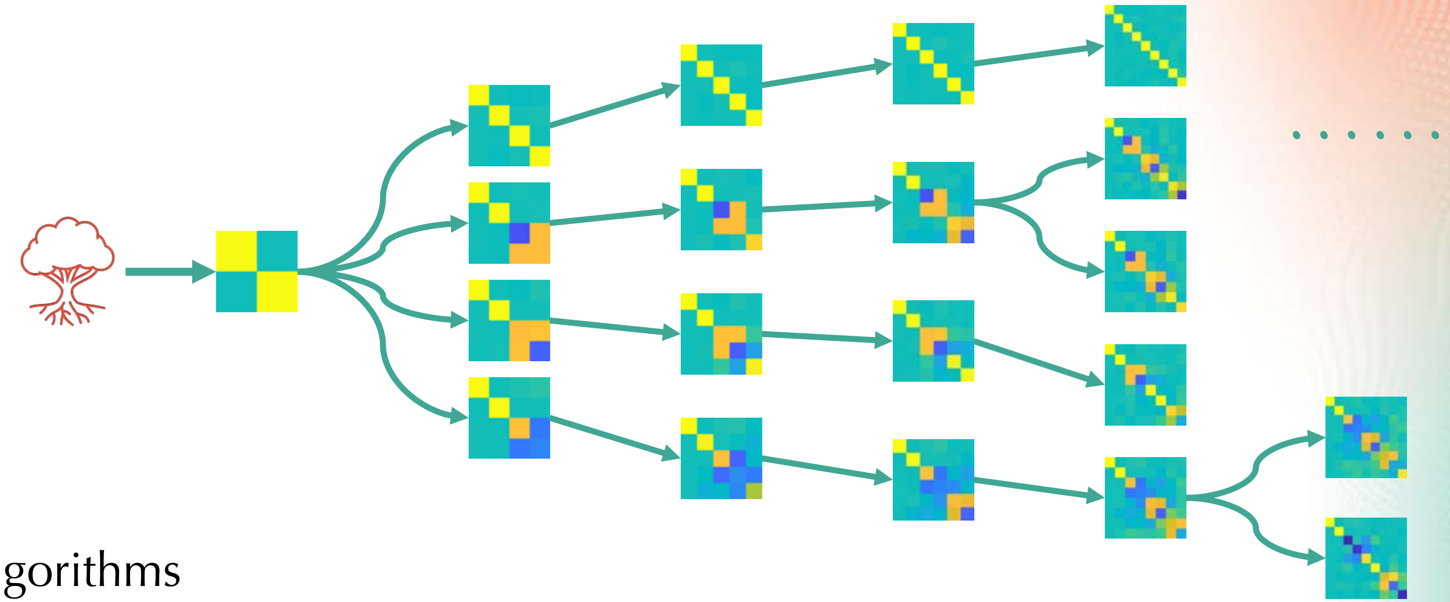


Functional map

Observations

- Intrinsic (global) symmetry information is encoded in the spectral domain of the shapes
- Pointwise maps can be organized in the spectral domain along the frequencies
- Use F-norm between two functional maps to approximate the distance between two pointwise maps

Map Tree Exploration



Algorithms
1. Functional map expansion rules
2. Map tree exploration

Map Tree Exploration

Contributions

- Progressive exploration of the map space
  - Start from the smallest functional map
  - Expand the functional map along the frequency domain
  - Only keep good & sufficiently different maps at each iteration

Map Tree Exploration

Contributions

- Output multiple maps, e.g.,
  - 4 maps for human v.s. human, human v.s. gorilla
  - 2 maps for animal shape pairs
  - ≥4 maps for man-made objects (table, knots, glasses, cup…)
- Previous work: a single map

Map Tree Exploration

Contributions

- Bijective ZoomOut for map refinement
  - ZoomOut [Melzi et al 2019]: spectral refinement method that enforces the orthogonality of a functional map
  - We propose Bijective ZoomOut that enforces both
    - Orthogonality of a single functional map
    - Bijectivity of the functional maps from both sides
  - Similar computation complexity
  - Better accuracy

Results

- Multi-solution shape matching
- Self-symmetry detection
- Non-rigid shape matching

Multi-solution shape matching

Source shape

Direct map | Symmetric | Back-to-front | Left-to-right & Back-to-front

Multi-solution shape matching

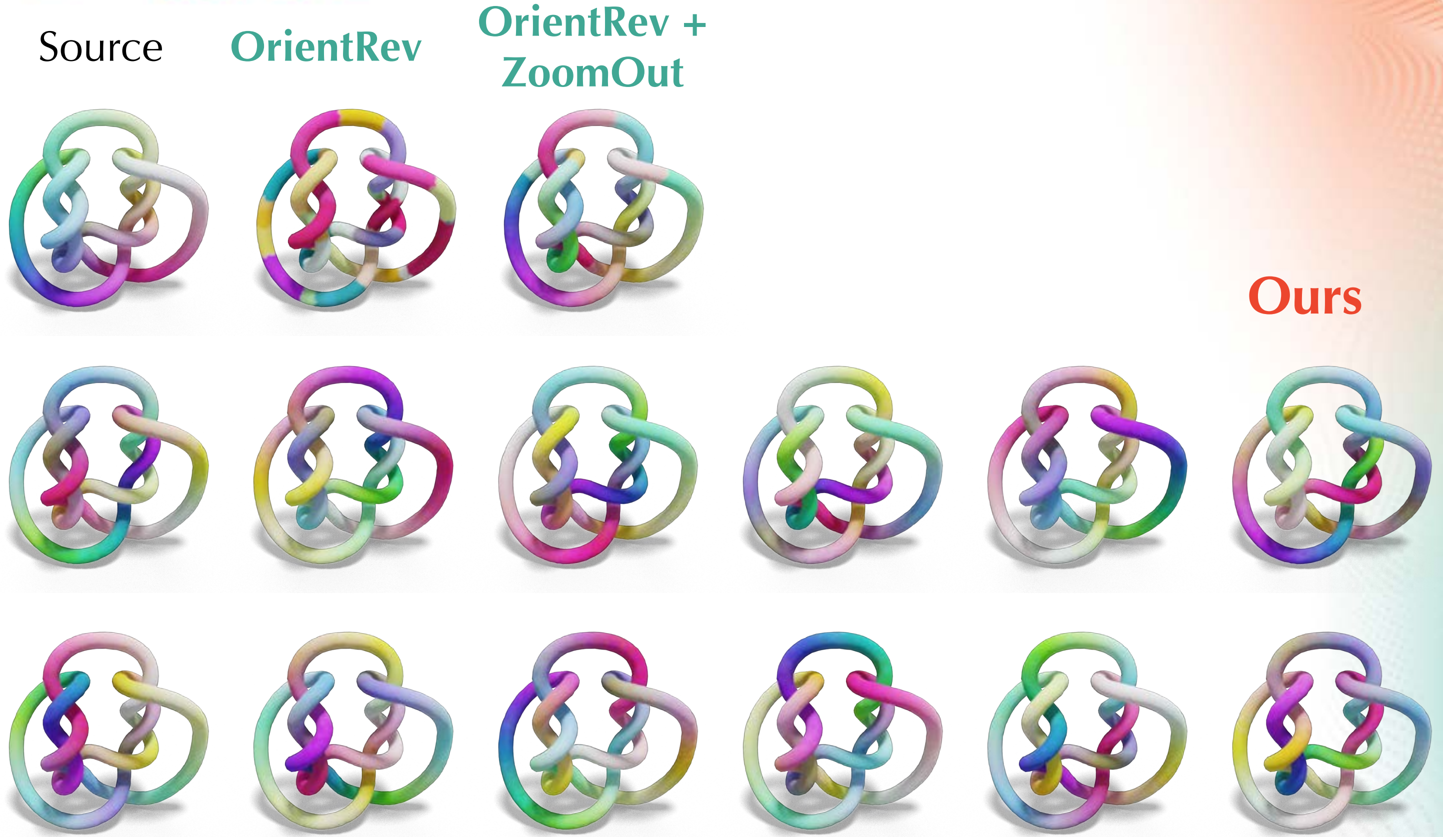
Source shape

Left-to-right

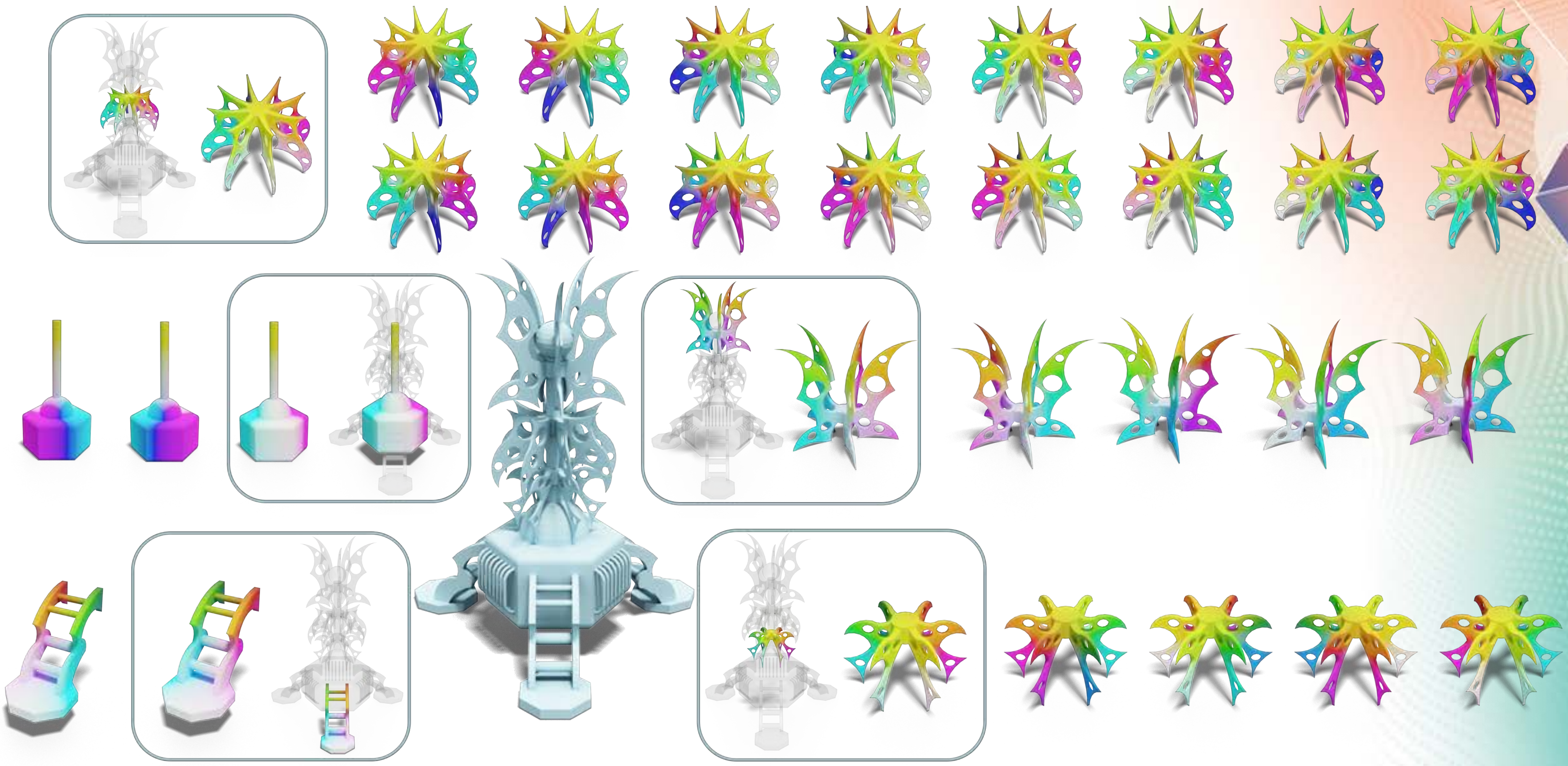
Upside-down

Left-to-right & Upside-down

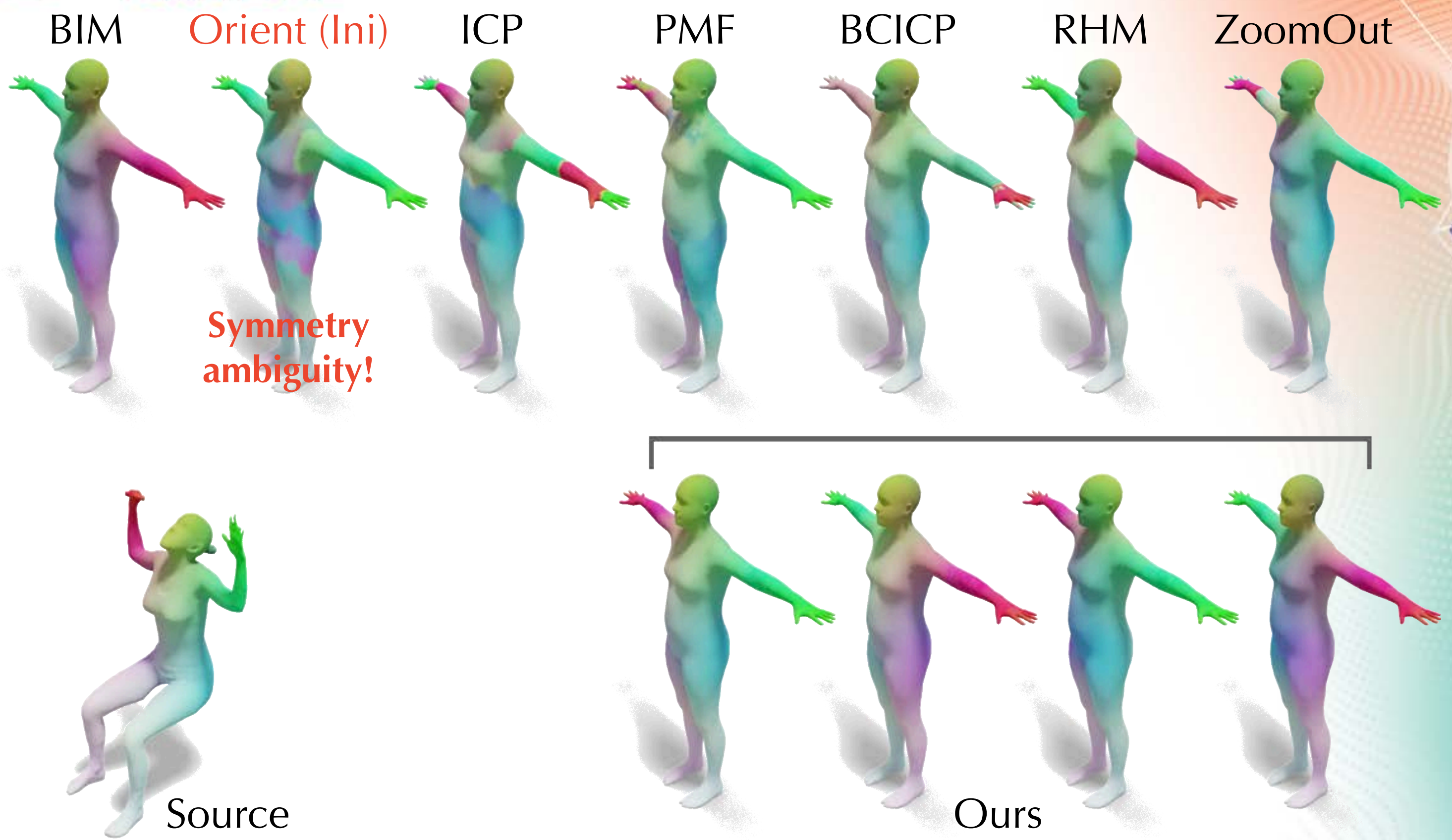
Multi-solution shape matching



Multi-solution shape matching



Non-rigid shape matching



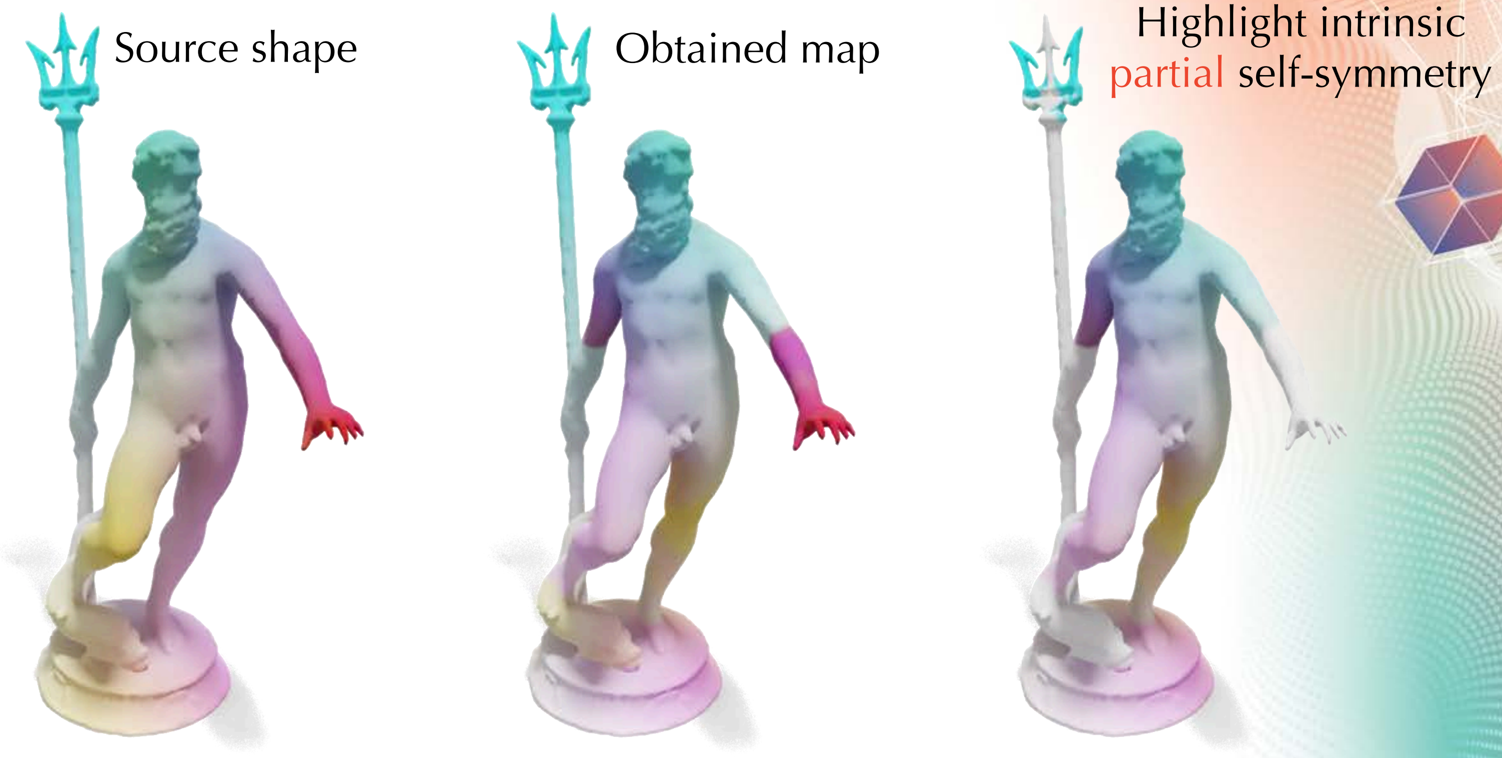
Non-rigid shape matching

SHREC'19 challenge: state-of-the-art accuracy

Methods \ Measurement	Accuracy ($\times 10^{-3}$)	GeoDist ($\times 10^{2}$)	Dirichlet Energy	Conformal Distortion	Runtime (sec)
BIM	83.69	**1.418**	**3.278**	1.970	164
GroupRep	311.1	5.254	13.41	7.787	3.95
IntSymm	**62.50**	1.945	12.17	7.123	2.05
OrientRev (Ini)	137.2	4.682	22.07	12.69	0.52
Ini + ICP	108.9	3.604	10.49	6.235	8.33
Ini + PMF	119.4	2.444	15.98	9.605	425
Ini + RHM	118.7	4.166	7.352	4.369	28.7
Ini + BCICP	96.72	2.466	5.633	3.741	157
Ini + ZoomOut	80.30	2.858	6.601	3.838	6.58
MapTree - GT	**39.62**	1.512	3.763	**0.949**	65.2
MapTree - Auto	**47.48**	1.507	3.833	**0.929**	65.2

We also propose an auto-selection algorithm via cycle-consistency!

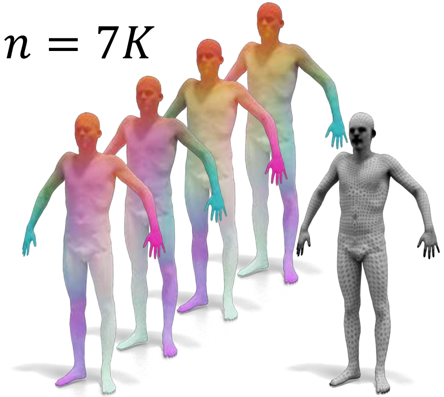
Self-symmetry detection



Robustness w.r.t. decimation

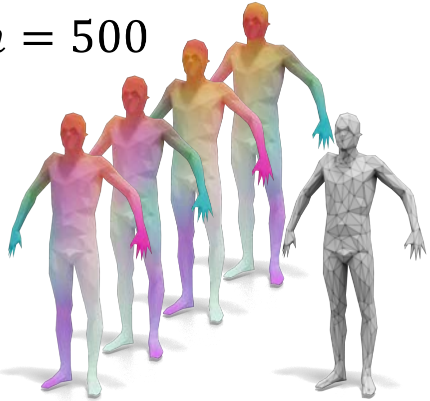
$n = 5K$

$n = 7K$

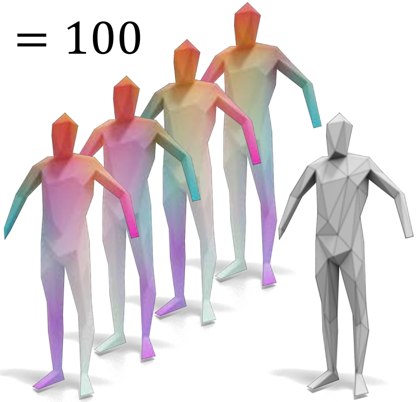


$n = 1K$

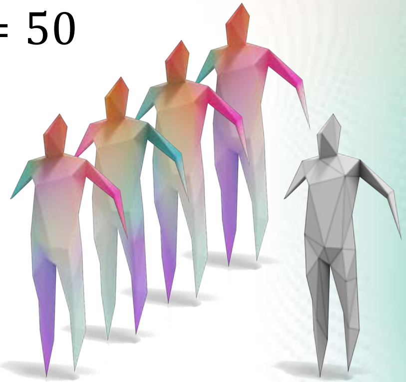
$n = 500$



$n = 100$



$n = 50$



Thank you for your attention ☺

MapTree:
Recovering Multiple Solutions in the Space of Maps

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Code:
https://github.com/llorz/SGA20_mapExplor